Polarized DIS in $\mathcal{N} = 4$ SYM: Where is spin at strong coupling?

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- J.H. Gao, BX, arXiv:0904.2870[hep-ph].
- Y. Hatta, T. Ueda, BX, arXiv:0905.2493 [hep-ph];

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Outline

1. AdS/CFT correspondence
   - Pedagogical introduction to AdS space
   - AdS/CFT correspondence

2. Polarized deep inelastic scattering and gauge/string duality
   - Large-x region
   - Small-x region
   - Spin puzzle

There are a few fundamental questions in spin physics:

- What can be said about the $\Delta\Sigma$ and ΔG in the strong coupling regime?
- Why is $\Delta\Sigma$ 'unnaturally' small, and what carries the rest of the total spin?
- How do the polarized parton densities and structure functions behave at small-x?

AdS/CFT can help to address and understand these questions.

- Using AdS/CFT, the strong coupling regime of $\mathcal{N} = 4$ SYM can be studied analytically.
- This might reveal some insights in QCD. AdS/CFT is a powerful tool. although nature might not have AdS.
- Why use AdS? String theory in flat spacetime does not work.
- There might be a conformal window in QCD. This may explain the form factor calculation.

However, one should keep in mind that

- QCD is not CFT, not $\mathcal{N} = 4$ SYM. CFT has no running coupling.
- $\mathcal{N} = 4$ SYM has no jets. [Hatta, Iancu, Mueller, 2008]

Outline

Anti de Sitter space 1

The AdS_5 space is a 5-dimensional hypersurface in 6 dimensions:

$$y_0^2 - y_1^2 - y_2^2 - y_3^2 - y_4^2 + y_5^2 = R^2$$

where R(not to be confused with $\mathcal{R}$) is called the radius of the AdS space.
Hyperbolic geometry (Constant negative curvature!) Change the coordinates as

$$\begin{aligned} y_0 &= \sqrt{R^2 + r^2} \sin \frac{t}{R}, \\ y_i &= r n_i \text{ with } i = 1, 2, 3, 4 \text{ and } \vec{n}^2 = 1 \\ y_5 &= \sqrt{R^2 + r^2} \cos \frac{t}{R}, \end{aligned}$$

Then the metric becomes,

$$\begin{aligned} ds^2 &= -dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 - dy_5^2, \\ &= -\left(1 + \frac{r^2}{R^2}\right) dt^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_3^2. \end{aligned}$$

The AdS_5 space is realized as the solution to the Einstein equation with a negative Λ_5.
For a AdS_5 black hole,

$$ds^2 = -\left(1 - \frac{\alpha_5 M}{r^2} + \frac{r^2}{R^2}\right) dt^2 + \frac{dr^2}{1 - \frac{\alpha_5 M}{r^2} + \frac{r^2}{R^2}} + r^2 d\Omega_3^2.$$

- Poincare Coordinates

$$
\begin{aligned}
r &= y_4 + y_5 , \\
x^{\mu} &= \frac{R}{r} \left(y_0 , y_1 , y_2 , y_3 \right) .
\end{aligned}
$$

Then the metric becomes,

$$ds^2 = \frac{r^2}{R^2} \left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \frac{R^2}{r^2} dr^2$$

Setting $z = R^2/r$, the metric becomes

$$ds^2 = \frac{R^2}{z^2} \left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2 \right)$$

$r = \infty$ or $z = 0$ is the Minkowski boundary.

- UV/IR correspondence[Susskind, Witten, 98].

$$E \sim \frac{r}{R^2}$$

Outline

AdS/CFT correspondence 1

Conjecture: $\mathcal{N} = 4$ Super Yang-Mills theory in $3+1$ dimensions

$\underbrace{\Leftrightarrow}_{\text{is the same as or dual to}}$ Type II B super string theory on $AdS_5 \times S^5$

$$ds^2 = \frac{R^2}{z^2}\left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2\right) + R^2 d\Omega_5^2$$

This is a solution to the Einstein equation in small r or large $z = R^2/r$ limit,

$$R_{\mu\nu} - \frac{\mathcal{R}}{2} g_{\mu\nu} = 8\pi T_{\mu\nu}, \qquad D_\nu F^{\mu\nu} = 0$$

where $T^{\mu\nu} = F_\mu^{\alpha\beta\gamma\delta} F_{\nu\alpha\beta\gamma\delta}$ and F_5 is called $R - R$ fields, which is generalization of $F_{\mu\nu}$.

$$\begin{array}{ccc} \text{Large 't Hooft limit in gauge theory} & \Leftrightarrow & \text{Small curvature limit in string theory} \\ g_{YM}^2 N_c \gg 1 & \Leftrightarrow & R^4/\alpha'^2 = R^4/l_s^4 \gg 1 \end{array}$$

$\mathcal{N} = 4$ Super Yang-Mills theory $\Leftrightarrow$ Type II B super string theory on $AdS_5 \times S^5$

$$\int \exp\left[iS_{4D} + \phi_0 \mathcal{O}\right] = \int_{AdS_5} \exp\left[iS_{5D}\right]$$

where S_{5D} contains non-trivial boundary condition $\lim_{z\to 0} \phi\left(\vec{x}, z\right) = \phi_0\left(\vec{x}\right)$. The correlation function of operators in 4D CFT is given by

$$\begin{aligned} \langle \mathcal{O}\left(x\right) \mathcal{O}\left(y\right)\rangle &= \frac{\delta}{\delta\phi_0(x)}\frac{\delta}{\delta\phi_0(y)}\langle e^{\int d^4x \mathcal{O}(x)\phi_0(x)}\rangle|_{\phi_0=0} \\ &= \frac{\delta}{\delta\phi_0(x)}\frac{\delta}{\delta\phi_0(y)} e^{-S_{\text{bulk}}[\phi_0]}|_{\phi_0=0} \end{aligned}$$

where $S_{\text{bulk}}[\phi_0]$ is the on-shell supergravity action in AdS_5 with boundary condition ϕ_0.

Field theory analogy(Harmonic oscillator):

$$\langle \mathcal{T} X(t_1) X(t_2) \rangle \propto \frac{\delta^2}{\delta J(t_1) \delta J(t_2)} e^{iS} \qquad \text{with} \quad S = \int dt (\frac{1}{2}\dot{x}^2 - \frac{1}{2} m x^2 + Jx)$$

Correspondence dictionary:

Gauge theory side (Operators)	Gravity side (Fields)
Operator $\mathcal{O}$	Dilaton ϕ
Energy momentum tensor $T_{\mu\nu}$	Graviton $h_{\mu\nu}$
Conserved current	Gauge field
....	....

Remark: Ads/CFT is a tool for computing correlation functions in strong coupling limit.

AdS/CFT correspondence 4

Conjecture: $\mathcal{N}=4$ Super Yang-Mills theory in $3+1$ dimensions

$\underbrace{\Leftrightarrow}_{\text{is the same as or dual to}}$ Type II B super string theory on $AdS_5 \times S^5$.

This conjecture is supported by many checks

- Symmetries: conformal symmetry $\Leftrightarrow$ isometry of AdS_5 and $SU(4)$ $\mathcal{R}$ symmetry $\Leftrightarrow$ isometry of S^5.
- Correlation functions: Some can be computed exactly in field theory and checked with AdS/CFT calculations.

Adding a black hole in AdS_5: the resulting metric becomes,

$$ds^2 = R^2\left[-h(u)dt^2 + \frac{du^2}{h(u)} + u^2(dx_1^2 + dx_2^2 + dx_3^2)\right] \quad \text{with} \quad u = \frac{1}{z}$$

where $h(u) = u^2\left[1 - \left(\frac{u_h}{u}\right)^4\right]$ and $u_h = \pi T$.

A few remarks:

- T is the Hawking temperature of this black hole.
- It will be identified as the temperature of the plasma.
- It breaks the conformal symmetry.

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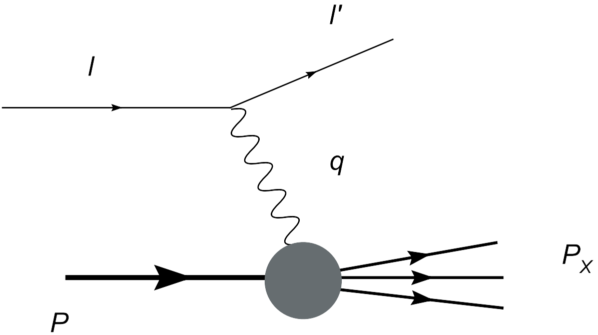
A gedanken experiment in gauge theories with large coupling.



Kinematic Variables

$$x = -\frac{q^2}{2p\cdot q} \quad \text{and} \quad q^2$$
$$M_x^2 = P_X^2 = (p+q)^2$$

The hadronic tensor $W^{\mu\nu}$ is defined as

$$W^{\mu\nu} = \int d^4\xi\, e^{iq\cdot\xi} \langle P, \mathcal{Q}, S|[J^\mu(\xi), J^\nu(0)]|P, \mathcal{Q}, S\rangle\,.$$

The hadronic tensor $W_{\mu\nu}$ can be split as

$$W_{\mu\nu} = W^{(\mathrm{S})}_{\mu\nu}(q, P) + i\, W^{(\mathrm{A})}_{\mu\nu}(q; P, S)\,.$$

Definition of structure functions and OPE

Assuming current conservation, $W_{\mu\nu}^{(\mathrm{S})}$ and $W_{\mu\nu}^{(\mathrm{A})}$ can be written as

$$
\begin{aligned}
W_{\mu\nu}^{(\mathrm{S})} &= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[F_1(x,q^2) + \frac{MS\cdot q}{2P\cdot q} g_5(x,q^2)\right] \\
&\quad - \frac{1}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2}\, q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\, q_\nu\right)\left[F_2(x,q^2) + \frac{MS\cdot q}{P\cdot q} g_4(x,q^2)\right] \\
&\quad - \frac{M}{2P\cdot q}\left[\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(S_\nu - \frac{S\cdot q}{P\cdot q}\, P_\nu\right) + \left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right)\left(S_\mu - \frac{S\cdot q}{P\cdot q}\, P_\mu\right)\right] \\
&\quad g_3(x,q^2)
\end{aligned}
$$

$$
W_{\mu\nu}^{(\mathrm{A})} = -\frac{M\,\varepsilon_{\mu\nu\rho\sigma}\, q^\rho}{P\cdot q}\left\{S^\sigma\, g_1(x,q^2) + \left[S^\sigma - \frac{S\cdot q}{P\cdot q}\, P^\sigma\right] g_2(x,q^2)\right\} - \frac{\varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma}{2P\cdot q} F_3(x,q^2).
$$

The OPE at large 't Hooft coupling

- Both in AdS/CFT and QCD, OPE is used to calculate structure functions.
- However, at large coupling, the physics is totally different. Only protected operators and double trace operators have finite anomalous dimensions.
- For operators which are not protected, their anomalous dimension is of order $\Delta \sim \tau \sim \gamma \sim \lambda^{1/4}$.
- Energy momentum tensor and conserved currents are protected operators.

[Polchinski, Strassler, 02],[Jianhua Gao, BX, 09]

- Break the conformal symmetry by introducing a confinement scale Λ.
- The current excites a gauge field $A_m$in 5D with a boundary condition $A_\mu(y,\infty) = n_\mu e^{iq\cdot y}$.
- The gauge fields satisfy 5D Maxwell equation and the solution is

$$
\begin{aligned}
A_\mu &= n_\mu e^{iq\cdot y}\frac{qR^2}{r}K_1(qR^2/r) \ , \\
A_r &= -iq\cdot n e^{iq\cdot y}\frac{R^4}{r^3}K_0(qR^2/r) \ .
\end{aligned}
$$

- The spin-$\frac{1}{2}$ hadron corresponds to supergravity mode of dilatino.
- The dilatino obeys 5D Dirac equation with the solution

$$
\psi = e^{ip\cdot y}\frac{C'}{r^{5/2}}\left[J_{\tau-2}(M_X R^2/r)P_+ + J_{\tau+1}(M_X R^2/r)P_-\right]u_\sigma \ ,
$$

- Supergravity approximation is valid when $\alpha'\tilde{s} = \frac{1}{\sqrt{\lambda}}\left(\frac{1}{x} - 1\right) \ll 1$, namely, $\frac{1}{\sqrt{\lambda}} \ll x < 1$. Thus only higher excitations are produced in the final state.

Structure functions

After computing

$$
\begin{aligned}
& n_\mu \langle P_X, X, \sigma' | J^\mu(0) | P, \mathcal{Q}, \sigma \rangle \\
= \;& i\mathcal{Q} \int d^6 x_\perp \sqrt{-g}\, A_m \bar{\lambda}_X \gamma^m \lambda_{\rm i} \\
= \;& i\mathcal{Q} \int d^6 x_\perp \sqrt{-g}\, \left(A_\mu \bar{\lambda}_X e^\mu{}_{\hat{\mu}} \gamma^{\hat{\mu}} \lambda_{\rm i} + A_r \bar{\lambda}_X e^r{}_{\hat{r}} \gamma^{\hat{r}} \lambda_{\rm i} \right)
\end{aligned}
$$

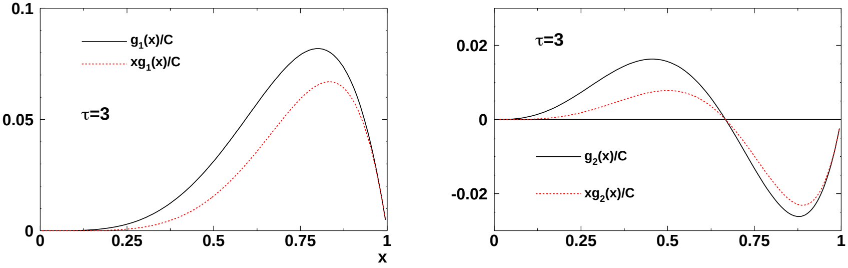
it is straightforward to read off the structure functions:

$$
\begin{aligned}
2F_1 &= F_2 = F_3 = 2g_1 = g_3 = g_4 = g_5 = \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2} \\
2g_2 &= \left(\frac{1}{2x} \frac{\tau+1}{\tau-1} - \frac{\tau}{\tau-1} \right) \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2}.
\end{aligned}
$$

Comments:

- In QCD, there is an interesting inequality $F_1 \geq g_1$. Here we see that $F_1 = g_1$, and the bound is saturated at finite x. However, at small-x, we find $F_1 > g_1$.
- The dilatino mode is chiral which gives nonzero parity violating structure functions.
- Double trace operators.

Plots of g_1 and g_2



- g_2 sum rule

$$\int_0^1 \mathrm{d}x g_2\left(x, q^2\right) = 0,$$

which is completely independent of τ and q^2. In QCD, this sum rule is known as the Burkhardt-Cottingham sum rule in large Q^2 limit.

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Small-x behavior of structure functions

[Y. Hatta, T. Ueda, BX, 09] We use wordsheet OPE approach to calculate small-x behavior of g_1 ($x \sim e^{-\sqrt{\lambda}}$). There are two protected operators in AdS/CFT.

- First one is energy momentum tensor $T^{\mu\nu}$, and it is dual to graviton with spin $j = 2$. $T^{\mu\nu}$ gives symmetric part of $W^{\mu\nu}$ and thus small-x contributions to F_1 and F_2.

  $$xF_1 \sim F_2 \propto x^{-1+2/\sqrt{\lambda})}$$

  Because of the curvature of the AdS space, the relevant value of j is shifted away from 2.

- The second one is conserved current J^μ, and it is dual to Kaluza-Klein photon with spin $j = 1$. The OPE of the current gives the antisymmetric part of $W^{\mu\nu}$

  $$\int d^4y\, e^{iqy} \langle PS|T\{J_3^\mu(y)J_3^\nu(0)\}|PS\rangle \Big|_{asym} = d^{33c}\epsilon^{\mu\nu}{}_{\alpha\beta}\frac{q^\alpha}{3P\cdot q}\frac{1}{x}\langle PS|J_c^\beta(0)|PS\rangle$$

  The imaginary part of above expression can be identified with structure functions.

The OPE approach

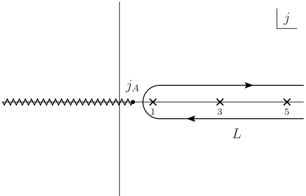
Let us focus on $\frac{1}{x}\langle PS|J_c^{\beta}(0)|PS\rangle$ which can be written as

$$\mathcal{Q}_c \int \frac{dj}{4i}\frac{1-e^{-i\pi j}}{\sin \pi j}\left(\frac{1}{x}\right)^j \int d^4y dz\sqrt{G}\int d^4y' dz'$$
$$\times \frac{1}{\Delta_j - 3 + 2(j-1)/\alpha'}\delta^{(5)}(u-u')J^{bulk}_{j+}(u')\bar{\psi}\gamma^+(\partial^+)^{j-1}\psi(z)$$

Remarks:

- $\int \frac{dj}{4i}\frac{1-e^{-i\pi j}}{\sin \pi j}$ ensures the sum over odd j values (same as in QCD).
- The t-channel propagator of exchanged KK photon satisfies 5D Maxwell equation, and its propagator is $\frac{1}{\Delta_j - 3+2(j-1)/\alpha'}$.
- Deforming the contour to the left and picking up the pole form the propagator, and choosing the imaginary part, it yields

$$g_1(x,Q^2) = F_3(x,Q^2) \sim \left(\frac{1}{x}\right)^{1-\frac{1}{2\sqrt{\lambda}}}\frac{e^{-(\rho-\rho')^2/4D\tau}}{\sqrt{\pi D\tau}}$$



$\tau = \ln 1/x$, $D = \frac{2}{\sqrt{\lambda}}$ and $\rho = \ln 1/z^2 \sim \ln Q^2$

g_1 is strongly peaked at $\tau \sim \frac{\sqrt{\lambda}}{2}\ln\frac{Q^2}{\Lambda^2}$

$\Leftrightarrow x \sim e^{-\sqrt{\lambda}}$.

Comparison between AdS/CFT and QCD

Table: Small-x behaviors of structure functions

	F_1	F_2	F_3	g_1^S	g_1^{NS}
AdS/CFT	$x^{-(2-\frac{2}{\sqrt{\lambda}})}$	$x^{-(1-\frac{2}{\sqrt{\lambda}})}$ [1]	$x^{-(1-\frac{1}{2\sqrt{\lambda}})}$	$\simeq 0$	$x^{-(1-\frac{1}{2\sqrt{\lambda}})}$ [2]
QCD	$x^{-(1+\frac{\ln 2}{\pi^2}\lambda)}$	$x^{-\frac{\ln 2}{\pi^2}\lambda}$ [3]	??	$x^{-2.5\frac{\sqrt{\lambda}}{2\pi}}$ [4]	$x^{-\frac{\sqrt{\lambda}}{2\pi}}$ [5]
Experiments	$x^{-1.08}$	$x^{-0.08}$	??	??	??

Comments:

- In AdS/CFT, F_1 and F_2 are calculated from reggeized graviton, while F_3 and g_1 arise from the t-channel exchange of a reggeized Kaluza-Klein photon.
- The singlet part of g_1 corresponds to non-conserved singlet current(hep-th/0104016). It has large anomalous dimension ($\gamma \simeq \lambda^{1/4}$) and vanishes in strong coupling limit.
- There might be continuous interpolation between the AdS/CFT and QCD when the t' Hooft coupling λ changes from ∞ to 0.

[1] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, [arXiv:hep-th/0603115].

[2] Y. Hatta, T. Ueda and B. W. Xiao, arXiv:0905.2493 [hep-ph].

[3] BFKL Pomeron

[4] J. Bartels, B. I. Ermolaev and M. G. Ryskin, arXiv:hep-ph/9603204.

[5] J. Bartels, B. I. Ermolaev and M. G. Ryskin, arXiv:hep-ph/9507271.

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The spin decomposition of a spin-$1/2$ fermion (e.g., proton or neutron)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L.$$

Table: Comparison between AdS/CFT and QCD

	$\Delta\Sigma$	ΔG	L
AdS/CFT	0	0	$1/2$
QCD	0.25	$\simeq 0$	large

Comments:

- In AdS/CFT, we find

$$\Delta\Sigma(Q^2) = \tilde{C}\left(\frac{\Lambda^2}{Q^2}\right)^{\lambda^{1/4}} \quad \text{and} \quad \Delta G(Q^2) = -\frac{\tilde{C}}{2}\left(\frac{\Lambda^2}{Q^2}\right)^{\lambda^{1/4}}.$$

[Gao, BX, 09], [Y. Hatta, T. Ueda, BX, 09]

- Bjorken sum rule:

$$\int_0^1 dx\, g_1(x,Q^2) = \frac{d^{33c}\mathcal{Q}_c}{12}A \quad \text{with} \quad \langle PS|J_c^\beta(0)|PS\rangle = \mathcal{Q}_c(AS^\beta + BP^\beta).$$

  A can be shown to be $F_1^5(0) = g_A$. We need to break chiral symmetry spontaneously ([hep-th/0306018]) and have massless pions to obtain nonzero A, otherwise, for example in hard wall model, it vanishes.

- g_2 sum rule (Burkhardt-Cottingham sum rule)

$$\int_0^1 \mathrm{d}x g_2\left(x, q^2\right) = 0,$$

  should be valid for all x from 0 to 1. This comes from Wandzura-Wilczek relation:

$$g_1(x,q^2) + g_2(x,q^2) = \int_x^1 \frac{dz}{z} g_1(z,q^2) + [\text{twist } 3]$$

  Note that [twist 3] contributions vanish due to large anomalous dimension, and $g_1(x,Q^2) \sim \frac{c}{x^{1-\epsilon}}$ together with $g_2(x) \sim -\frac{c}{1-\epsilon} + \frac{\epsilon c}{(1-\epsilon)x^{1-\epsilon}}$.

Summary

- Small-x behavior of polarized structure functions at strong coupling.
- Bjorken sum rule and Burkhardt-Cottingham sum rule are valid also in AdS/CFT.
- The entire hadron spin may come from orbital momentum at strong coupling.